# Understanding the Quantum Mechanism of the Hydrogen Spectrum through Fourier Series Energy Field Analysis

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摘要：在前面论文中，我们给出了相对论方程质速关系的一个新的傅立叶级数表达式。根据物质波理论，利用傅立叶级数形式的量子力学的能量场公式，给出了量子隧道效应、氢原子光谱等实验现象一种新的量子机制解释，推导得出经典的的里德伯公式仅仅是我们新公式的一个近似简化形式，与传统的量子力学理论和公式相比，其数理逻辑更完善更简单，也更准确，对于其他系列量子力学实验现象也可以给出合理和更符合物理理论数学逻辑的解释。

**Abstract:** In the previous paper, we gave a new Fourier series expression of the relativistic equation mass-velocity relation. According to the matter wave theory, a new quantum mechanism explanation of quantum tunneling effect, hydrogen atom spectrum and other experimental phenomena is given by using the energy field formula of quantum mechanics in the form of Fourier series. It is deduced that the classical Rydberg formula is only an approximate simplified form of our new formula. Compared with the traditional quantum mechanics theory and formula, its mathematical logic is more perfect, simpler and more accurate. It can also give a reasonable and more consistent explanation to other series of quantum mechanics experimental phenomena.

**Key words:** theoretical physics; Fourier series; Quantum mechanics; Hydrogen spectrum; Quantum phenomenon

## 0.Preface

In the previous paper, we gave the Fourier series expression for mass-velocity relation of the relativistic formula, then introduced the matter wave theory and obtained a new quantum mechanical expression. One of the main achievements of quantum mechanics is the explanation of quantum tunneling effect, hydrogen atom spectrum, which will be explained by a new Fourier series expression of energy field or mass field.

## 1.The analyses and discussion

In the derivation of Einstein's relativistic formula, the most important transformation is the Lorentz transform, which involves the mass-velocity relation coefficient, $1/\sqrt{1\text{-}v^2/c^2}$ where v is the velocity of the object, c is the speed of light. We can get the Fourier series form expression of the mass-velocity relation in previous paper[1-13].

$$f(x)=\frac{m_0^2}{m^2}=\frac{2}{3}-\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi\cos\frac{nv\pi}{c} \quad (1)$$

For the motion process, we can give the for speed differential of (14) equation, multiplied i can obtained the following complex variable function:

$$\frac{df(x)}{dv}i=\sum_{n=1}^{\infty}\frac{c}{n\pi}\frac{\partial\frac{m_0^2}{m^2}}{\partial v}i=i\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi\sin\frac{nv\pi}{c} \quad (2)$$

According to Euler's formula:

$$\frac{m_0^2}{m^2}-\sum_{n=1}^{\infty}\frac{c}{n\pi}\frac{\partial\frac{m_0^2}{m^2}}{\partial v}i=\frac{2}{3}-\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi e^{\frac{nv\pi}{c}i} \quad (3)$$

Because the static energy $E_0$ and relativistic energy E, the relationship between the two is also a mass-velocity relationship:

$$\frac{E_0^2}{E^2}=\frac{m_0^2c^4}{m^2c^4}=\frac{m_0^2}{m^2} \quad (4)$$

Therefore the substituting (4) into (3) gets:

$$E^2\text{-}E_0^2=\frac{1}{3}E^2+E^2\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi\cos\frac{nv\pi}{c} \quad (5)$$

This is the energy field equation in the steady state of uniform velocity (5).

## 2. quantum tunneling effect

quantum tunnelling effect

The famous quantum tunneling effect in quantum mechanics can be well explained by ( 4 ) and ( 5 ). Since the distribution of energy e belongs to an attenuation wave process, its energy particles are always in a state of fluctuation with high energy and low energy due to fluctuation. therefore, some high energy particles can penetrate the potential well, showing quantum tunneling effect. since E has maximum value, there is a limit for tunneling of quantum potential well, and it is related to propagation distance and initial energy.

The real equation also includes the dynamic energy release of the matter wave (6).

$$\frac{E_0^2}{E^2}-\sum_{n=1}^{\infty}\frac{c}{n\pi}\frac{\partial\frac{E_0^2}{E^2}}{\partial v}i=\frac{2}{3}-\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi e^{\frac{nv\pi}{c}i} \qquad (6)$$

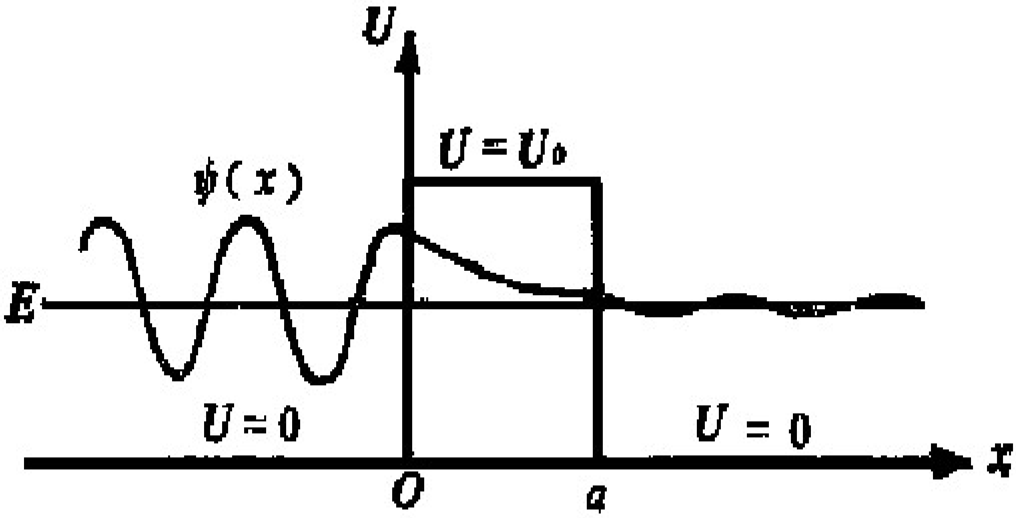


Fig.1 The schematic of quantum tunneling effect

3. Theoretical Explanation of hydrogen atomic spectrum

We know that quantum mechanics has a perfect theoretical explanation for the spectrum of hydrogen atoms, so can our formula explain it well? For light wave or electromagnetic wave close to the speed of light C, its own propagation will naturally form energy wave motion. Only at a certain value can there be an integer solution, which corresponds to the lowest energy value. The energy value of photon is discrete, and E=hv, V corresponds to spectra of different wavelengths.

Then it is obtained from ( 5 ) and ( 6 ). Because the dynamic process and the steady process are different, they are discussed separately. Steady state process:

Then from (5) (6) get , because motion would change the process.

$$E_1^2\text{-}E_0^2=\frac{1}{3}E_1^2+E_1^2\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi\cos\frac{nv_1\pi}{c} \qquad (7)$$

$$E_2^2\text{-}E_0^2=\frac{1}{3}E_2^2+E_2^2\sum_{m=1}^{\infty}\frac{4}{m^2\pi^2}\cos m\pi\cos\frac{mv_2\pi}{c} \qquad (8)$$

(8) - (7) :

$$\frac{2}{3}(E_2^2\text{-}E_1^2)=E_2^2\sum_{m=1}^{\infty}\frac{4}{m^2\pi^2}\cos m\pi\cos\frac{mv_2\pi}{c}\text{-}E_1^2\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi\cos\frac{nv_1\pi}{c} \qquad (9)$$

$$\frac{2}{3}(E_2\text{-}E_1)=\frac{E_2^2}{E_2+E_1}\sum_{m=1}^{\infty}\frac{4}{m^2\pi^2}\cos m\pi\cos\frac{mv_2\pi}{c}\text{-}\frac{E_1^2}{E_2+E_1}\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\cos n\pi\cos\frac{nv_1\pi}{c}=\frac{2}{3}\mathrm{h}\nu \qquad (10)$$

For a static spectrum, the distribution of the spectrum is based on the difference of $1/m^2$ and $1/n^2$. The triangular function series is convergent, and its value can be approximately considered to be determined by several differences such as 1, 1 / 4, 1 / 9, 1 / 16, 1/25($1/m^2$- $1/n^2$ ). Therefore, the spectrum of hydrogen atoms can also be determined by the (10) fromula. It is also equivalent to the Bohr's theory explanation of modern quantum mechanics. The explanation of experimental phenomena is more in line with mathematical logical.

When the speed v approaches the speed of light c， $E_2$-$E_1$ and（$1/m^2$ -$1/n^2$）are approximately proportional，the hν is related to（$1/m^2$ -$1/n^2$）approximately also, hν=hc/λ, so 1/λ is related to（$1/m^2$ -$1/n^2$）approximately. (10 )is simplified to obtain(11).

$$1/\lambda=R\ (1/m^2-1/n^2) \qquad (11)$$

the simplified formula ( 11 ) is equivalent to the Rydberg formula(12). Different values give other spectra of hydrogen atoms, which can well explain the spectra of Lehmann, Balmer and so on.

$$1/\lambda=R\ (1/n^2-1/n_1^2) \quad (n_1=n+1) \qquad (12)$$

The spectrum should be discrete, because the values of $E_1$ and $E_2$ can only be specific values and are discontinuous.

Because v=Δl/Δt, and the distance can be calculated using spatial position coordinates,

$$\Delta \mathrm{l}=\sqrt{(x1-x)^2+(y1-y)^2+(z1-z)^2} \qquad (13)$$

Or

$$\Delta \mathrm{l}=\sqrt{r_2^2+r_1^2-2r_1r_2(\cos\theta_1\cos\theta_2\cos(\phi_2-\phi_1)+\sin\theta_1\sin\theta_2)} \qquad (13)$$

Therefore, equation ( 9 ) can obtain the derivative for the distance coordinate parameter, obtain the optimal value solution of minimum energy value, and then obtain the stable orbital radius value of the hydrogen atom.

Planck's constant h can be considered a coincidence expressed by equation ( 10 )[14-16]. The 2 / 3 approximation is just close to Planck's constant value. If the electron mass and velocity (motion energy) is substitute in the (10), the solution value is just close to Planck's constant value.

## 4.Conclusion

a. The energy field equation formula we deduced contains the summation term of trigonometric series. The equation itself has wave characteristics, and the dynamic energy field or mass field is an attenuated matter wave field. Moreover, its solution shows that it is discrete, which is consistent with the particle characteristics. The equation itself can well explain wave-particle duality.
b. The new equation can well explain the existing experimental phenomena of quantum mechanics. due to wave-particle duality, it can well explain the existing experimental phenomena such as quantum tunneling effect and one-dimensional harmonic oscillator. It is more in line with mathematical logic than the existing theories.
c. The new energy field or mass field equation, comparison and analysis of its solution show that the explanation of hydrogen atomic spectrum is completely combined with Barmore formula and is more accurate, and Barmore formula is its simplified approximate form. Existing known test data fully support the new equation.

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